**Checking Valid Coupon Codes**

**Problem Statement:** Checking Valid Coupon Codes Using HashSet in C#

**Objective**: Write a C# program to validate coupon codes entered by users in an e-commerce system using a `HashSet` to efficiently check if the entered code exists in a predefined list of valid coupon codes.

**Description**:

You are developing an e-commerce application where users can apply coupon codes to receive discounts. The system maintains a predefined list of valid coupon codes, and users submit codes to check if they are valid. Your task is to use a `HashSet` to store the valid coupon codes and verify whether a user-entered coupon code is valid. The program should prompt the user for a coupon code, check if it exists in the `HashSet`, and display an appropriate message indicating whether the coupon is valid or invalid. The solution should be case-insensitive to ensure flexibility for user input.

**Requirements**:

1. Use `HashSet<string>` to store the predefined list of valid coupon codes.

2. Prompt the user to enter a coupon code.

3. Convert the user input to a consistent case (e.g., uppercase) for comparison.

4. Use the `HashSet`’s `Contains` method to check if the entered coupon code is valid.

5. Display a message indicating whether the coupon is valid or invalid.

**Input**:

- A `HashSet<string>` of valid coupon codes, e.g., { "SAVE10", "FREESHIP", "DISCOUNT20" }.

- A user-entered coupon code (string) via console input, e.g., "save10" or "WELCOME".

**Output**:

- A message indicating whether the coupon code is valid or invalid.

- Example output:

Enter your coupon code:

SAVE10

Coupon is valid! Applying discount...

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Enter your coupon code:

WELCOME

Invalid coupon code.

**Constraints**:

- Coupon codes are strings and should be validated case-insensitively (e.g., "SAVE10" and "save10" are considered the same).

- The valid coupon code list may vary in size, including being empty.

- The solution must use `HashSet` for efficient lookup of coupon codes.

**Learning Goals:**

- Understand how to use `HashSet` in C# for fast membership testing.

- Learn to handle user input and perform case-insensitive comparisons.

- Practice using `HashSet`’s `Contains` method for real-time validation.

- Explore a practical application of `HashSet` in an e-commerce scenario for validating user inputs like coupon codes.